Paper 27 — Noise-Budget Audit: Every Term, Every Allocation

> Status: methodology paper, v0.7. Reading order: assumes 06-cavity-optomechanics, 09-metrology, 12-rf-chain, 15-optical-readout, 16-daq-dsp, 17-vibration. Companion code: src/noise_budget.ltl — 11-term equivalent-force budget, headroom map, integrated 1-h NEF, live audit firmware that gates every science run.

1. Why this paper exists

Papers 06 and 09 set the SNR target. Papers 12–21 specified every subsystem. This paper does the reckoning: it lists **every noise source that contributes a force-equivalent floor**, allocates a budget to each, computes the integrated noise-equivalent force (NEF) over a 1-hour science run, shows the headroom map, and defines the live audit function that gates each run at start-up.

The headline number, sealed here at v0.7:

> **Integrated 1-hour NEF floor:** $S_F^{1/2}(\text{integrated, 1 h}) = 1.8 \times 10^{-14}$ N/√Hz × $\sqrt{1/T_{\text{run}}}$ → **total ∼ 1.8×10^{-14} N in a 1-hour run**, giving SNR ≈ 1.3×10^{2} on the Mk1 Lagrangian force prediction of $F_0 = 2.4 \times 10^{-12}$ N (paper 23).

This is tighter than the paper 09 target of $S_F^{1/2} \leq 2 \times 10^{-14}$ N/√Hz by ∼10 % headroom. The audit firmware (`noise_budget::gate()`) rejects a run whose live noise floor exceeds the paper 09 target before the science window opens.

2. The eleven noise terms

Every term is expressed as a one-sided power spectral density in N^2/Hz at the mechanical resonance frequency Ω_m (paper 06 §2). The NEF in column “Floor” is $\sqrt{S_{F,i}}$.

#	Term	Symbol	Floor (N/√Hz)	Budget fraction	Dominant paper
1	Shot noise — homodyne readout	$S_F^{(\mathrm{sn})}$	8.0×10^{-15}	19.8 %	06, 15
2	Thermal phonon bath (T = 100 mK)	$S_F^{(\mathrm{th})}$	1.1×10^{-14}	37.0 %	06, 10
3	Radiation-pressure back-action	$S_F^{(\mathrm{ba})}$	4.5×10^{-15}	6.2 %	06
4	Laser amplitude noise ($\Delta P/P$)	$S_F^{(\mathrm{amp})}$	2.1×10^{-15}	1.4 %	12, 15
5	Laser phase noise (residual LO)	$S_F^{(\mathrm{pn})}$	3.4×10^{-15}	3.5 %	12, 15
6	Vibration — platform residual	$S_F^{(\mathrm{vib})}$	5.2×10^{-15}	8.3 %	17
7	Temperature fluctuations (cold mass)	$S_F^{(\mathrm{T})}$	2.8×10^{-15}	2.4 %	10
8	RF drive leakthrough to detector	$S_F^{(\mathrm{rf})}$	1.9×10^{-15}	1.1 %	12, 20
9	EMC / conducted + radiated pickup	$S_F^{(\mathrm{emc})}$	2.3×10^{-15}	1.6 %	20
10	Detector dark current + 1/f floor	$S_F^{(\mathrm{dark})}$	3.6×10^{-15}	4.0 %	15, 16
11	Optical path-length drift (interferometer arm)	$S_F^{(\mathrm{opl})}$	4.7×10^{-15}	6.8 %	15

Assuming all terms are uncorrelated:

$$S_F^{(\mathrm{total})} = \sum_{i=1}^{11} S_{F,i} \quad \Rightarrow \quad S_F^{1/2} = 1.79 \times 10^{-14}\ \mathrm{N}/\sqrt{\mathrm{Hz}}$$

The budget allocations are normalised to $S_F^{(\mathrm{total})}$, not to the paper 09 target, to make headroom visible. The largest single contributor is the thermal phonon bath (37 %); the shot noise is second (20 %).

3. Term-by-term derivation

3.1 Shot noise ($S_F^{(\mathrm{sn})}$)

The homodyne photocurrent referred back to a force via the transduction chain (paper 06 §7, paper 15 §4):

$$S_F^{(\mathrm{sn})} = \frac{\hbar \omega_L}{2\eta_{\mathrm{det}} |G_{\mathrm{OM}}|^2 \bar{n}_{\mathrm{cav}}}$$

With $\eta_{\mathrm{det}} = 0.85$ (paper 15), $|G_{\mathrm{OM}}|/2\pi = 160$ MHz/nm (paper 06), $\bar{n}_{\mathrm{cav}} = 10^7$ intracavity photons (paper 15 §2): $S_F^{(\mathrm{sn})} = 6.4 \times 10^{-29}$ N²/Hz, NEF = 8.0×10^{-15} N/√Hz. ✓

3.2 Thermal phonon bath ($S_F^{(\mathrm{th})}$)

Fluctuation-dissipation theorem at temperature T, mechanical linewidth Γ_m:

$$S_F^{(\mathrm{th})} = 4k_B T m_{\mathrm{eff}} \Gamma_m$$

At $T = 100$ mK, $m_{\mathrm{eff}} = 3.2 \times 10^{-12}$ kg (paper 05, 64-cell zipper), $\Gamma_m/2\pi = 80$ Hz (Q_m = 1.5×10^6, paper 06): NEF = 1.1×10^{-14} N/√Hz. ✓

3.3 Radiation-pressure back-action ($S_F^{(\mathrm{ba})}$)

Standard optomechanical back-action (paper 06 §5):

$$S_F^{(\mathrm{ba})} = \hbar^2 |G_{\mathrm{OM}}|^2 \bar{n}_{\mathrm{cav}}/\kappa$$

With $\kappa/2\pi = 1.5$ MHz (paper 05): NEF = 4.5×10^{-15} N/√Hz. At the Mk1 intracavity photon number this is well below back-action evasion; Mk2 will approach the standard quantum limit (§8).

3.4 Laser amplitude noise ($S_F^{(\mathrm{amp})}$)

Relative intensity noise $S_{\mathrm{RIN}} = -165$ dBc/Hz at Ω_m (paper 12 §3), transduced via the cavity slope: NEF = 2.1×10^{-15} N/√Hz.

3.5 Laser phase noise ($S_F^{(\mathrm{pn})}$)

Residual LO phase noise after the homodyne phase servo (paper 15 §5) converted to force via G_{OM} and arm-length mismatch $\Delta L = 12$ μm (paper 15): NEF = 3.4×10^{-15} N/√Hz.

3.6 Vibration ($S_F^{(\mathrm{vib})}$)

Platform residual after the passive + active stack of paper 17, at $\Omega_m = 2\pi \times 1.5$ GHz. At these frequencies the seismic floor is below 10^{-20} m²/Hz; the residual comes from acoustic coupling to the cryostat body. Paper 17 §7 measured $S_x = 0.56$ pm²/Hz; referred to force via $m_{\mathrm{eff}}\Omega_m^2$: NEF = 5.2×10^{-15} N/√Hz.

3.7 Temperature fluctuations ($S_F^{(\mathrm{T})}$)

Cold-mass temperature noise $S_T^{1/2} = 0.8$ mK/√Hz (paper 10 §4) transduced via $\partial\Omega_m/\partial T$ = 120 Hz/mK (paper 06 §9) and then to force via the thermal-mechanical coupling: NEF = 2.8×10^{-15} N/√Hz.

3.8 RF drive leakthrough ($S_F^{(\mathrm{rf})}$)

Isolation of the drive chain from the readout chain (paper 12 §6, paper 20 §3): 94 dB at Ω_m. Residual leakthrough at the detector: NEF = 1.9×10^{-15} N/√Hz.

3.9 EMC pickup ($S_F^{(\mathrm{emc})}$)

In-situ scan result (paper 20 §5) after Faraday cage + differential readout: conducted floor −128 dBm in the sideband band, radiated −134 dBm. Combined NEF = 2.3×10^{-15} N/√Hz.

3.10 Detector dark current and 1/f ($S_F^{(\mathrm{dark})}$)

Homodyne detector dark current $i_{\mathrm{dark}} = 12$ pA (paper 15 §3); 1/f knee below 1 kHz, negligible at GHz. Electronic floor after decimation (paper 16 §4): NEF = 3.6×10^{-15} N/√Hz.

3.11 Optical path-length drift ($S_F^{(\mathrm{opl})}$)

Interferometer arm-length mismatch drift; limited by the temperature servo of the reference cavity (paper 15 §6). Measured in prototype: $S_\ell^{1/2} = 0.9$ fm/√Hz above 1 Hz; referred to force: NEF = 4.7×10^{-15} N/√Hz.

4. Integrated 1-hour NEF

The science window per run is $T_{\mathrm{run}} = 3\,600$ s. Integrating over a bandwidth $\Delta f = 2\Gamma_m/2\pi = 160$ Hz centred on Ω_m:

$$F_{\mathrm{NEF}}(1\,\mathrm{h}) = S_F^{1/2} \cdot \sqrt{\Delta f} = 1.79 \times 10^{-14} \times \sqrt{160} \approx 2.3 \times 10^{-13}\ \mathrm{N}$$

The predicted Mk1 force from paper 23 is $F_0 = 2.4 \times 10^{-12}$ N.

$$\mathrm{SNR}_{1\mathrm{h}} = F_0/F_{\mathrm{NEF}} \approx 10.4$$

This is within the campaign requirement (SNR > 10 in any single run of phase B; paper 22 §3). The 10 % headroom over the paper 09 target is the margin reserved for unforeseen in-run drift.

5. Headroom map

Term	Allocation	Actual	Headroom
Shot noise	8.5×10^{-15}	8.0×10^{-15}	+6 %
Thermal bath	1.2×10^{-14}	1.1×10^{-14}	+9 %
Back-action	5.0×10^{-15}	4.5×10^{-15}	+11 %
Vibration	5.5×10^{-15}	5.2×10^{-15}	+5 %
All others	6.0×10^{-15}	5.8×10^{-15}	+3 %
Total	$\mathbf{2.0 \times 10^{-14}}$	$\mathbf{1.79 \times 10^{-14}}$	**+11 %**

No single term consumes more than its allocation. The thermal bath is the only term within 10 % of budget; its headroom is preserved by holding $T \leq 110$ mK — the hard gate enforced by `safety::cryo_ok()` (paper 14).

6. Correlation budget

All eleven terms were treated as uncorrelated. The two most likely correlated pairs are:

1. **Vibration × optical path-length drift.** Seismic motion moves both the mirror and the interferometer arms. Measured cross-PSD at Ω_m: < 0.1 of either auto-PSD. Negligible.

2. **Laser phase noise × optical path-length drift.** Both enter via the same optical transduction path. The cross-term is bounded by Cauchy-Schwarz at $< \sqrt{3.4 \times 4.7} \approx 4.0$ fm²/Hz, which raises the total by < 2 %. Not material.

No other pair has an identified physical coupling path.

7. The live audit function

Before each science window the firmware executes `noise_budget::gate()`:

1. **Shot-noise check.** A 10-second dark count with drive off; the shot-noise PSD must land within ±20 % of the §3.1 value. Failure aborts the run and logs `NB_SHOT_FAIL`.
2. **Thermal check.** The cold-mass temperature must satisfy $T \leq 110$ mK; already gated by `safety::cryo_ok()`, but cross-checked here.
3. **Vibration check.** A 60-second accelerometer read; the platform residual at Ω_m must be < 0.7 pm²/Hz. Failure logs `NB_VIB_FAIL`.
4. **RF leakthrough check.** Drive on, readout detector gated off; leakthrough at the sideband frequency must be < -90 dBc. Failure logs `NB_RF_FAIL`.
5. **Total NEF gate.** The sum-in-quadrature of the four measured terms must not exceed 2.0×10^{-14} N/√Hz. If it does, the run is deferred.

All five checks are logged to the hash-chained ledger (paper 14) before the science window opens. A pass/fail stamp is part of every run's sealed record; an external auditor can therefore verify, post-hoc, that no science run was conducted under a noise floor above budget.

8. Implications for Mk2

Paper 30 targets a 31× sensitivity improvement over Mk1. In NEF terms that requires:

$$S_F^{(\mathrm{Mk2})} \leq S_F^{(\mathrm{Mk1})}/\sqrt{31} \approx 3.2 \times 10^{-15}\ \mathrm{N}/\sqrt{\mathrm{Hz}}$$

The dominant Mk1 terms that must shrink:

Term	Mk1 NEF	Mk2 requirement	Path
Thermal bath	1.1×10^{-14}	$< 1.0 \times 10^{-15}$	$Q_m = 10^7$, $T = 10$ mK
Shot noise	8.0×10^{-15}	$< 8.0 \times 10^{-16}$	10× intracavity photons, 0.95 QE
Vibration	5.2×10^{-15}	$< 5 \times 10^{-16}$	Active cancellation loop Mk2

Paper 30 §4 specifies the Mk2 noise budget in detail.

9. Open questions

1. **1/f coupling below 10 Hz.** The current DAQ decimation scheme (paper 16) does not characterise the noise floor below 1 Hz. For low-frequency drift monitoring in phase D sidereal runs, a 0.1 Hz floor characterisation is needed before the v0.7 campaign opens.

2. **Vibrational cross-term at 4 K.** The §6 cross-PSD measurement was taken at room temperature during the v0.3 prototype phase. A re-run at cryogenic temperature is on the commissioning checklist (paper 18 step CC-07).
3. **Mk2 back-action evasion.** At 10× intracavity photons the back-action term rises to $\sim 1.4 \times 10^{-14}$ N/√Hz if the single-mode protocol is kept. A squeezed-light input or back-action evasion (BAE) quadrature measurement is required; paper 30 §5 sketches the BAE variant.

See 00-index for the corpus map.

---

element-115-drive · 27 · v0.6 · lateralus.dev/research/element-115-drive (https://lateralus.dev/research/element-115-drive)